

LXXIV. *An Explanation of an Instrument for measuring small Angles, the first Account of which was read before the Royal Society May 10, 1753. By Mr. John Dollond. In a Letter to James Short, M. A. and F. R. S.*

S I R,

Read Apr. 25, 1754. **T**HE account which I gave you, some time ago, of a new micrometer, was contained in as few words as possible; being rather desirous, that experiments might be made, before I said much concerning it: But since your many repeated experiments have confirmed what was expected from it, I have endeavoured to draw up a more full account of this instrument, with demonstrations of the principles which it is founded upon, which I here send you inclos'd, and which you may lay before the Royal Society, if you think proper. I am, Sir,

Your most obedient humble servant,

Denmark-Court,
April 4. 1754.

John Dollond.

BEfore I enter upon particulars relating to this micrometer, it will be proper to make a few preparatory observations on the nature of spherical glasses, so far as may be necessary to render the following explanation more easily understood.

Observation 1. It is a property of all convex spherical glasses to refract the rays of light, which are transmitted through them, in such a manner, as to collect all those that proceed diverging from any one point

point of a luminous object, to some other point; whose distance from the glass depends chiefly on its convexity, and the distance of the object from it.

Obs. 2. The point, where the rays are thus collected, may be considered as the image of that point, from which they diverge. For if we conceive several radiant points thus emitting rays, which, by the refractive quality of the glass, are made to converge to as many other points; it will be an easy matter to understand, how every part of the object will be truly represented. As this property of spherical glasses is explained and demonstrated by all the writers on optics; it being the very foundation of the science, the bare mention of it is sufficient for the present purpose.

Obs. 3. It will be necessary, however, to observe farther, that the lines connecting every point in the object with its corresponding ones in the image, do all intersect in a certain point of the axis or line passing thro' the poles of the glass, where its two surfaces are parallel, and may be properly called its centre: Whence it appears, that the angles subtended by the object and its image from that point, must be equal; And therefore their diameters will be in the same ratio, as their distances from that point.

Obs. 4. As the formation of the image by the glass depends entirely on the property above-mentioned, that is to say, its collecting all the light, that is incident on it, from the several points of the object into as many other points at its focus; it follows, that any segment of such a glass will also form an image equal, and every way similar, to that exhibited by the whole glass; with this difference only, that it will be so much darker, as the area of the segment is less than that of the whole glass.

Obs.

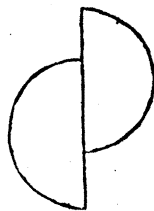
Obs. 5. The axis of a spherical glass is a line connecting the centres of the spheres, to which the two surfaces are ground; and where-ever this line passes thro' the glass, there the surfaces are parallel. But if it happens, that this line does not go thro' the substance of the glass, such a glass is said to have no internal centre; but it is conceived to be in its plane produced, till it meets the axis: And this imaginary point, tho' external to the glass, is as truly its centre, and is as fixed in its position to it, as if it were actually within its substance.

Obs. 6. If a spherical glass, having its center or pole near its middle or centre of its circumference, should be divided by a strait line through the middle; the centre will be in one of the segments only. For how exact soever a person may be supposed to be in cutting it thro' the centre; yet 'tis hard to conceive, how a mathematical point should be divided in two: Therefore the centre will be internal to one of the segments, and external to the other. But if a small matter be ground away from the strait edge of each segment, both their centres will become external; and so they will more easily be brought to a coincidence.

Obs. 7. If these two segments should be held together, so as to make their centres coincide; the images, which they give of any object, will likewise coincide, and become a single one. This will be the case, when their strait edges are joined to make the glass, as it were, whole again: But let the centres be any-how separated, their images will also separate, and each segment give a separate and distinct image of any object, to which they may be expos'd.

Obs. 8. Tho' the centres of the segments may be drawn from their coincidence, by removing the seg-

ments in any direction whatever; yet the most convenient way for this purpose is, to slide their strait edges one along the other, till they are remov'd, as the figure in the margin represents them: For thus they may be moved without suffering any false light to come in between them. And by this way of removing them, the distance between their centres may be very conveniently measured; *viz.* by having a Vernier's division, commonly, tho' falsely, call'd a Nonnius's, fixed to the brass-work, that holds one segment, so as to slide along a scale on the plate, to which the other part of the glass is fitted.



Obs. 9. As the images of the same object are separated, by the motion of the segments; so those of different objects, or different parts of the same object, may be made to coincide. Suppose the sun, moon, or any planet, to be the object; the two images thereof may, by this contrivance, be removed, till their opposite edges are in contact: In which case, the distance between the centres of the two images will be equal to the diameter of either; and so of any other object whatever.

Obs. 10. This divided glass may be used, as a micrometer, three different ways. In the first place, it may be fixed at the end of a tube, of a suitable length to its focal distance, as an object-glass; the other end of the tube having an eye-glass fitted as usual in astronomical telescopes. Secondly, It may be applied to the end of a tube much shorter than its focal distance, by having another convex glass within the tube, to shorten the focal distance of that, which is cut in two. Lastly, It may be applied to the open end of a reflecting

flecting telescope; either of the Newtonian, Gregorian, or Cassegrain construction. And tho' this last method is much the best, and most convenient, of the three; yet, as the first is the most natural, as well as the easiest to be understood; it will be proper to explain it fully, and to demonstrate the principles, on which this micrometer is constructed, by supposing it made use of in the first way: Which being done, the application of it to other methods will be readily understood.

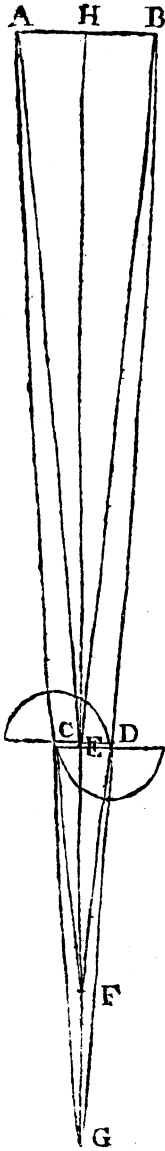
Having thus, by the foregoing observations, given a general idea of the nature and effects of this divided object glass, I shall proceed to demonstrate the principles, from whence the measures of the angles are to be obtained by this instrument; which will be done by the following propositions.

PROPOSITION I.

Suppose a divided object-glass fixed at the end of a tube, according to the first method, and the tube directed to the object intended to be measured; and suppose, likewise, the segments removed from their original position, in the manner directed under Obs. 8. till the opposite edges of the two images are seen in contact at the focus of the eye-glass: Then, I say, the angle subtended, by the distance between the centres of the segments, from the focus of the eye-glass, where the edges are seen in contact, is equal to the angle subtended by the diameter of the object from that same point.

Demonstration.

Let the line AB (see the next page) represent the diameter of the object to be measured; and the



points CD the centres of the two glass segments : Also G the focus where the images of the extremities of the object are coincident. It is evident, from *Obs.* 3. that AG and BG are straight lines, that pass thro' the centres of the segments, and connect the extreme points of the object with their corresponding points in the images ; and therefore, as the diameter of the object and the distance between the centres of the segments are both inscribed between these two lines, they must needs subtend the same angle from the point where those lines meet ; which is at G .
Q. E. D.

The focal distance CG , or DG , is variable, according to the distance of the object from the glass : So that it decreases as the distance of the object from the glass increases ; and when the object is so far off, that the focal length of the glass bears no proportion to its distance ; then will it be least of all, as CF or DF ; and the point F is call'd the focus of parallel rays. Any other focus, as G , being the focus of a near object, is call'd a respective focus ; as it respects a particular distance : But the focus of parallel rays respects all objects that are at a very great distance ; such as is that of all the heavenly bodies.

PROPOSITION II.

The distance HE of the object from the glass, is to EF, the focal distance of parallel rays, as the distance HG of the object from its image is to EG, the distance of the image from the glass: That is, HE : EF :: HG : EG.

The demonstration of this proposition may be gathered from any treatise of dioptrics; it being a general rule for finding the respective focus to any given distance, when the focus of parallel rays is known.

PROPOSITION III.

The angle subtended by the diameter of the object, from the glass, is equal to that subtended, by the opening of the centres of the segments, from the focus of parallel rays. That is, the angle AEB equal to the angle CFD.

Demonstration.

It appears, by inspection of the figure, that $AB : CD :: HG : EG$.

And by the last proposition $HE : EF :: HG : EG$.

Then, as the two last terms of these two analogies are alike; the two first terms of one will be in the same proportion as the two first terms of the other; which gives the following proportion: $AB : CD :: HE : EF$. Whence the truth of the proposition is evident.

From

From this proposition it appears, that the angle subtended by the diameter of the object from the glass, is found without any regard to the distance of the object, or to the distance of the respective focus, where the image is seen; as the measure depends entirely upon the focus of parallel rays and the opening of the segments. We may likewise, from hence, derive a rule for the quantity of the angle, without considering the length of the glass. Let an object, whose diameter is known, be set up at some known distance; the angle it will subtend from the glass may then be found by trigonometry: Then let it be measured by this micrometer, and the distance, between the centres of the segments, found on the scale already mentioned, will be the constant measure of the same angle, in all other cases: Because the distance of the object makes no alteration in the measure of the angle, as has been demonstrated: And thus having obtain'd the distance between the centres of the segments, which answers to any one angle, all other distances may be computed by the rule of three.

All that has been hitherto said relates to the first method of using this micrometer; that is, by fitting it to the end of a tube suited to its focal length, and by viewing the images with a proper eye-glass, in the manner of an astronomical telescope. But the length of the tube, in this way, would be very troublesome; and therefore it will be proper to consider other methods, for an easier management. I shall, therefore, proceed to the second method, mention'd in *Obs.* 10. which is, by using another object-glass to shorten the focus of that which serves for the micrometer. To faci-

facilitate the understanding of this method, it will be necessary to premise the following observation.

Obj. II. Rays of light, which are brought to such convergency as to form the image of an object, proceed, after that, diverging, in the manner they did when they issued from the object before they were transmitted thro' the glass; and therefore they may be again collected by another spherical glass, so as to form a second representation of the same object; which may again be repeated by a third glass, &c. So that the first image may be consider'd as an object to the second glass and the second image will be an object to the third, and so on. Tho' these images may be very different, in respect to their magnitudes, yet they will be all similar; being true representations of the same object: This will hold good, tho' the second glass should be put so near the first as to receive the rays before the image is form'd: For as the rays are tending to meet at a certain distance, the second will receive them in that degree of convergency, and, by an additional refraction, bring them to a nearer focus; but the image will still be similar to that which would have been made by the first glass, if the second had not been there.

Upon this principle all refracting telescopes are made; some of which are a combination of four, five, or six, glasses. The first glass forms an image of the object; the second repeats the image, which it receives from the first; and so on, till the last glass brings a true representation of the object to the eye. The same may be said of reflecting telescopes: For a spherical mirror acts in the same manner, in that respect, as a spherical glass.

Now

Now let this be applied to the subject in hand. Suppose the focal distance of the divided object-glass to be about forty feet; and suppose the segments to be open'd wide enough to bring the opposite edges of an object in contact: Then let another object-glass, uncut, be fix'd within the tube, of a proper degree of convexity, to shorten the focus of the other as much as may be requir'd; suppose to twelve feet: By what has been just now observ'd, this glass will present the two images in the same form which would have been exhibited by the divided glass, if this other glass had not been there. For tho' the images are not yet form'd, when the second glass receives the rays; yet, as those rays are converging towards it, the second glass must represent those images in the same position, and form, as the tendency of the rays requires. For while the segments are fix'd in their position to each other, their images will also be fix'd in their position; and let them be repeated ever so many times, by refraction thro' spherical glasses, or by reflexion from spherical mirrors, they can suffer no alteration in their position to one another. By this means, the telescope may be shorten'd, at pleasure, tho' the scale for the measure of the angles will remain the same. The only inconvenience, which the shortness of the telescope introduces, is a want of sufficient distinctness; which will so far hinder the exactness of the observation, as the contact of the edges cannot be so accurately determin'd, as they might be with longer telescopes.

This difficulty is intirely remov'd by fixing the divided glass at the end of a reflecting telescope: For the reflexions and refractions, which the rays must undergo

undergo in passing thro' the telescope, will no way alter the position of the images which the rays, that have passed thro' the segments, are tending to : For, as has been already observ'd, a number of reflexions and refractions may repeat the images, and alter their magnitudes ; but can make no alteration in their proportions.

Therefore this way of fixing the divided glass to a reflecting telescope, which was the third method propos'd, is, by far, the best ; as such telescopes of moderate and manageable lengths, when well made, are capable of magnifying considerably, and shewing objects to great advantage. This micrometer's being applicable to the reflecting telescope, with so much certainty, is no inconsiderable advantage : For any one will easily understand, that, to measure the diameter of a planet exactly, it is necessary, that the planet be magnify'd, and shew'd distinctly, which could not be obtain'd, in the common way, without very great lengths ; such as render'd it very difficult, not to say impracticable, to take exact measures. Besides, the common micrometer is limited, in this respect, upon another account ; *viz.* because the diameter of the planet cannot be measur'd, without having the whole planet within the field of the telescope which confines the magnifying power within very narrow bounds ; whereas, by this method, nothing more is requir'd, than to see the contact of the edges, which allows the magnifying power to be increas'd at pleasure.

In the common micrometer, the object is to be taken between two wires, so that the contact of its edges with those wires cannot be observ'd at one view ;

and the least motion of the telescope, whilst the observer is turning his eye from one wire to the other, must oblige him to repeat the observation; whereas, by this method the contact of the edges of the images is not at all affected by the motion of the telescope. Whence the comparison of this micrometer, with the common sort, in this respect, stands thus: The one requires great steadiness in the telescope, but yet it is applicable to none, but such as are very difficult to keep steady; the other does not require such steadiness, tho' it is applicable to short telescopes, which are easily managed.

These advantages not only add to the certainty of the observation, but assist vastly in the expedition; for an observer may make twenty observations, in this way, where he could scarce, with much fatigue, be sure of one with the common micrometer. Expedition in making observations, must be allowed a very great advantage, in this climate, where the uncertainty of the weather renders astronomical observations so precarious, that no opportunities, even the most transient, should be let slip. An instance of this was given in to the Royal Society, in an account of the eclipse of the sun last October.

As the motion of the telescope gives the observer no great inconvenience, in this method; neither does the motion of the object at all disturb his observation (I mean such a motion, as that of the heavens is). This gives him leave to take the diameter of a planet, in any direction; or the distance between two stars or planets, let their situation be how it will; in which respect the common micrometer is absolutely defective; as it can give no angles, but such as are perpendicular

pendicular to the line of their motion ; tho' the diameters of the planets, in other directions, is very much wanted ; it being highly probable, from the laws of motion, and what we see in Jupiter, that such planets, as revolve round their axes, have their polar diameters shorter than their equatorial ones.

The distances of Jupiter's satellites from one another, or from Jupiter's body, cannot be measured, with any certainty, in the common way, as their position is always very far from being at right angles with the line of their motion : Neither can the moon's diameter, which must be taken from horn to horn, scarce ever be obtained that way, because it very rarely happens, that the diameter, to be measured, lies at right angles to the line of her motion. The same may be said of the distance between two stars. But this micrometer gives angles, in every direction, with equal ease and certainty ; the observation being also finished in an instant, without any trouble or fatigue to the observer. For as there are no wires made use of, this way, in the field of the telescope ; so the observer has no concern about any illumination. The largeness of the scale deserves also to be taken notice of, as it may, in this micrometer, be increas'd almost at pleasure, according as the smallness of the object requires. Another inconvenience attending the common micrometer is, the variation of the scale, according to the distance of the object. As the telescope must be lengthen'd, or drawn out farther, for short distances ; the scale, which depends upon that length, is thereby increas'd ; which renders the measure of the angle very uncertain : Whereas, in this micrometer, the scale is the same at all distances ; so that the angle may be measured with the utmost certainty,

tainty, without any regard to the distance of the object.

Upon the whole, it may be concluded, that this micrometer is a complete instrument in its kind ; having many advantages above the common sort, without any of their disadvantages : And there is no doubt, but, when brought into practice, it will tend much to the advancement of astronomy.

LXXV. *An Account of an Earthquake felt at York on the 19th of April 1754. In a Letter from Mr. David Erskine Baker to Tho. Birch, D. D. Secret. R. S.*

Dear Sir,

Read Apr. 25, 1754. **O**N Friday night last, the 19th instant, at about eleven o'clock, we were alarm'd, in this city, with the shock of an earthquake. As I was myself in London, and felt both the shocks, which happen'd there in the year 1750, I became immediately sensible of what it was. In the room, in which I was sitting, which was on a first floor, the tremulous, or rather undulating, motion of the floor was very plain ; and the windows rattled, as if they had been shaken by a sudden squall of wind. The shock lasted for about three seconds, and was attended, or rather preceded, by a rumbling noise, not much unlike that made by an empty hearse driven over a stone pavement, and, indeed, exactly the same with that, which I remember to have heard, with both the shocks, in London, in 1750. The violence of the shock